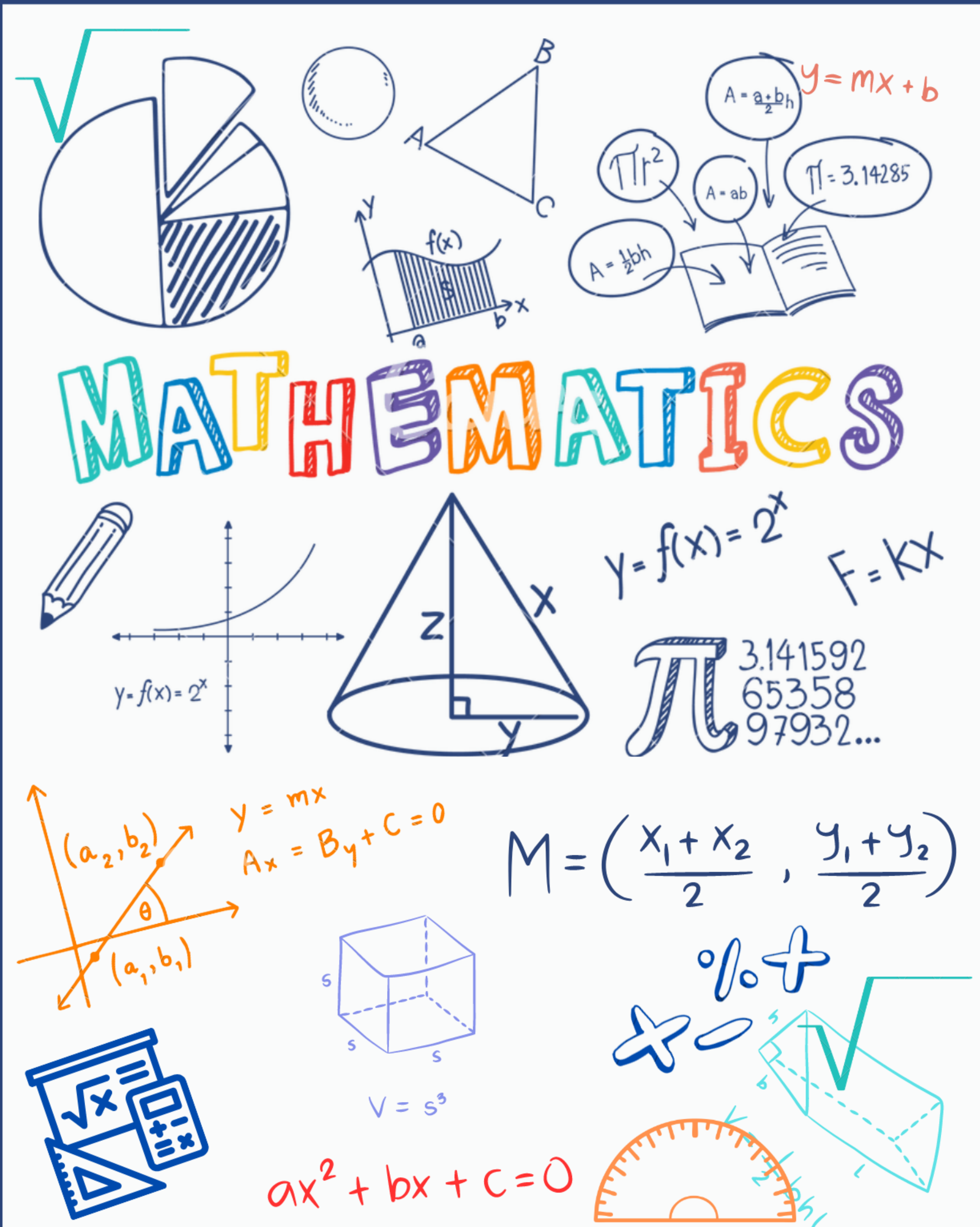
# AMaThing

DEPARTMENT OF MATHEMATICS AND HUMANITIES
SARDAR VALLABHBHAI NATIONAL INSTITUTE OF TECHNOLOGY




ISSUE #4 MARCH 2023

# Message from HoD

**Dr. Jayesh M. Dhodiya**

It is my pleasure to release AMaThing 4.0 on the International Day of Mathematics, which is observed on March 14. As India has taken over G20's presidency, I believe that Mathematics can unite the world, therefore creating "Vasudhaiva Kutumbakam."

In the present time, mathematics is applicable far beyond what we had ever imagined. It should be our responsibility to analyze the nature and create nearly perfect mathematical models to represent them which will help the world in reducing chaos.

Researchers should have a clear mission and vision while presenting their research in layman's terms. Even among us, we should start thinking about interdisciplinary research that will further unify humankind and create better solutions for future generations.

At the end, I would like to thank the dedicated, hard-working team that has laboriously produced this wonderful document.

Wishing the best
Dr. Jayesh M. Dhodiya

# Message from the Editorial Team

Dear authors and readers

We are honoured to be a part of the publication of this issue of AMaThing. Recently, we have diversified our field of acceptance, and this issue has seen a significant increase in articles that relate to science communication, which might bring huge success to this issue.

We had gone through tough times to bring this scholastic issue this time. We have raised the bar in accepting the articles and laid down stringent rules in publishing articles that have delayed the release of the issue.

The team wishes to excel in the magazine articles by diversifying and accepting articles related to further allied fields of mathematical sciences in order to meet global standards. We will continue to publish quality articles that may help reduce the vast expanse of science and society.

We assure the readers that they will have a joyful and thought-provoking time reading through the issue.

Cheers.

Ekata Jain
On behalf of the Editorial Team, 2022-23.

# Contents

# How a Research Idea Explores in Science Community

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*We often think of discovery or invention as a process in the mind of a singular genius. A flash of inspiration strikes and —eureka!— suddenly, we get a new way to describe dark matter or the equations for gravity or mind-blowing fact about the universe. However, the real deal is with the idea, how it travel from one brilliant mind to the other minds. In this article, I want to show you a theoretical simulation work on how a research idea explores the science community.*

"*I learned very early the difference between knowing the name of something and knowing something.*"
Richard Feynman

## Intuition

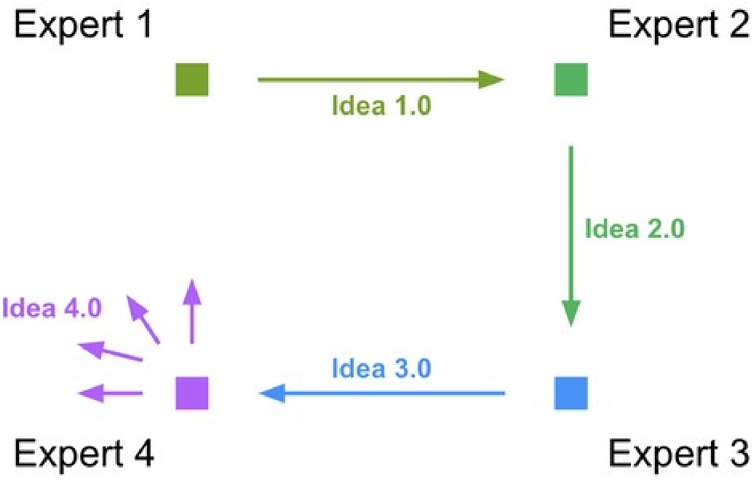
Let us start with how exactly the basic setup of any simulation (model) works in the mathematical world. Almost every simulation work is a primary or modified version of the SIR (Susceptible, Infectious, Recovered) model. In the science community, susceptibles are people with the same (or maybe different) research interests. There are no such things as infectious or recovered people in the science world; one needs to draw an analogy here to understand better. I am not a modelling expert, so I would be very cautious before generalising any of the lessons here without deep consideration. I believe it is healthy to engage our little scientists in experiments and research, especially if the alternative is uncertain.

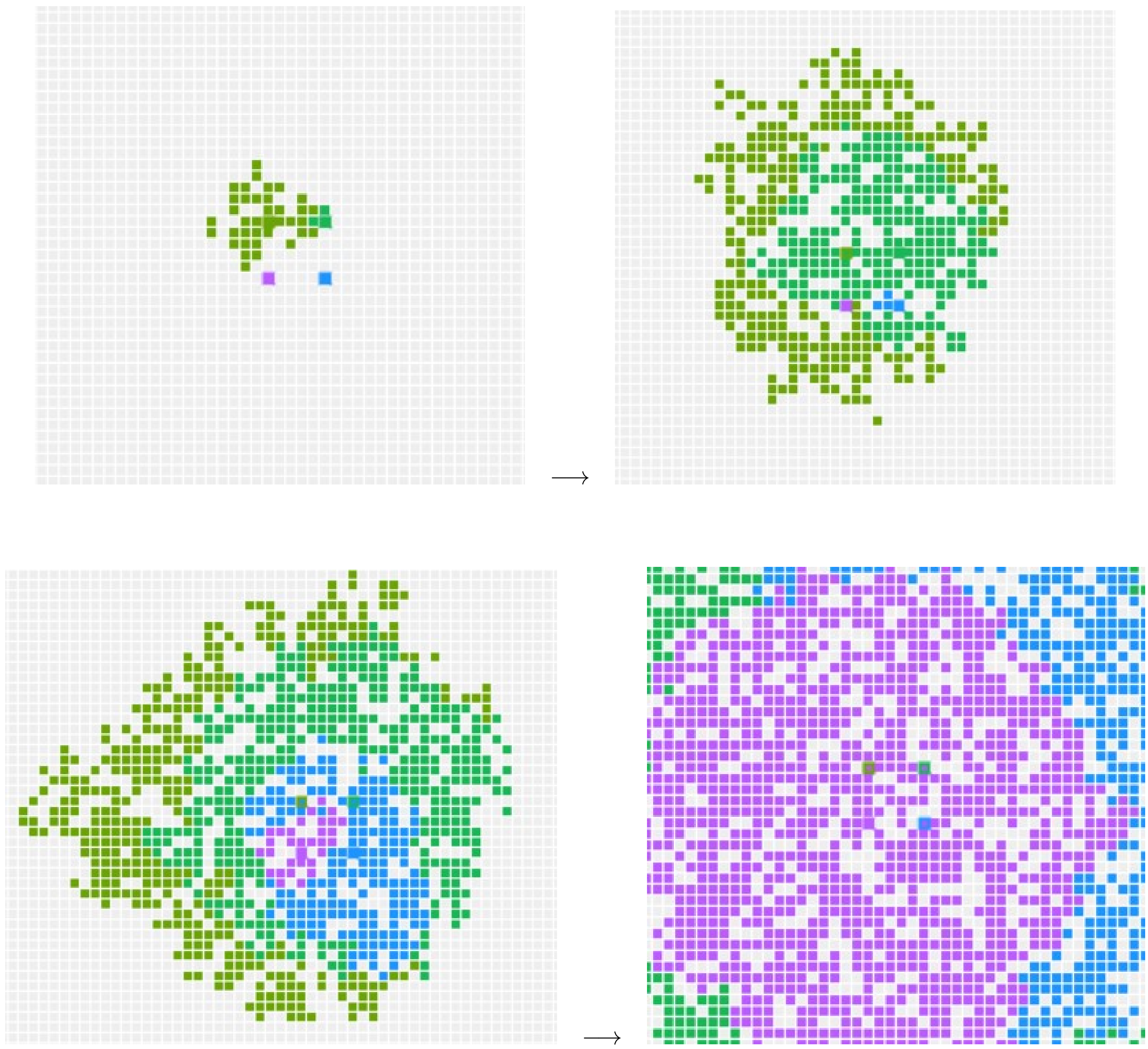
Let us start simple with the layout, and if required, gradually make things more complex.

## Network and Its Density

Usually, with properly constructed networks, the invention is something that happens on a network. Beginning with the importance of a network and its density, a network is essential in two ways. First, preexisting approaches have to make their way into the mind of the inventor (these are the citations of a new paper, the bibliography section of a new book, the giants on whose shoulders' Newton and Euler stood). Second, the network is essential for getting a new idea back out into the world (an invention that does not spread is hardly worth calling an "invention" at all). Therefore, the spread of knowledge is a diffusion process. Take the example of the discovery of gravity. Newton established gravity, having been hit by an apple on his head, as a force of attraction due to masses. After a period, Einstein developed a whole new theory of gravity. Here, the idea was passed from one genius to another in order to make it more correct and less uncertain. Generalise this concept with many experts (say four), and each one of them knew how to transform the previous idea into a better version of itself. Nevertheless, here the speed of spread of an idea also depends on how "close" they are (in the sense of the same period or how dense their network is), and an expert as a member of the International Education Society always has a slightly greater chance of encountering a new idea (or transforming it into a more desirable version).

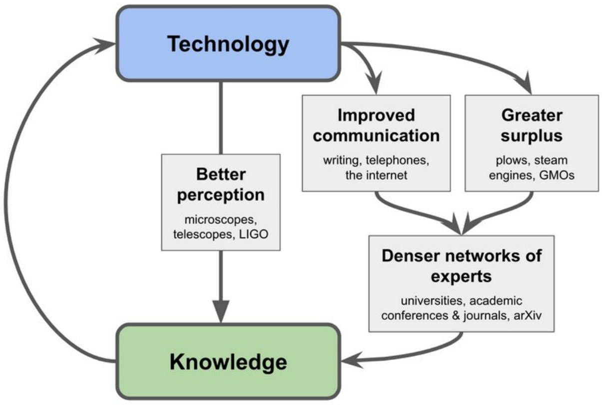
I will present rough simulation images of how knowledge might diffuse and grow within a network with conventional density.



♠ *Here, each white square-grid represents susceptible people*
♠ *The conceit of this simulation is that we need all four experts to contribute to the final version of the idea. And at each phase of development, the idea has to diffuse to the relevant expert.*

# Knowledge and Technology




In the scientific community, there is a never-ending loop within knowledge and technology. We use new knowledge to devise new tools. For example, understanding the physics of semiconductors equips us to develop powerful computers. Similarly, already known knowledge leads us to build new technology; this new technology gives us new data, intuition, and results to extend the domains of our knowledge. We have a well-known example in the Large Hadron Collider (LHC), the world's largest and highest-energy particle collider. Over 10,000 scientists worked on it. LHC helps us with the completeness of the Standard Model of physics. New technologies, especially in travel and communication, change the structure of the social networks on which knowledge grows. For example, over the last two decades, the number of users on ResearchGate increased from 25,000 to more than 17 million. In particular, it allows experts and specialists to network more tightly with one another (network density).

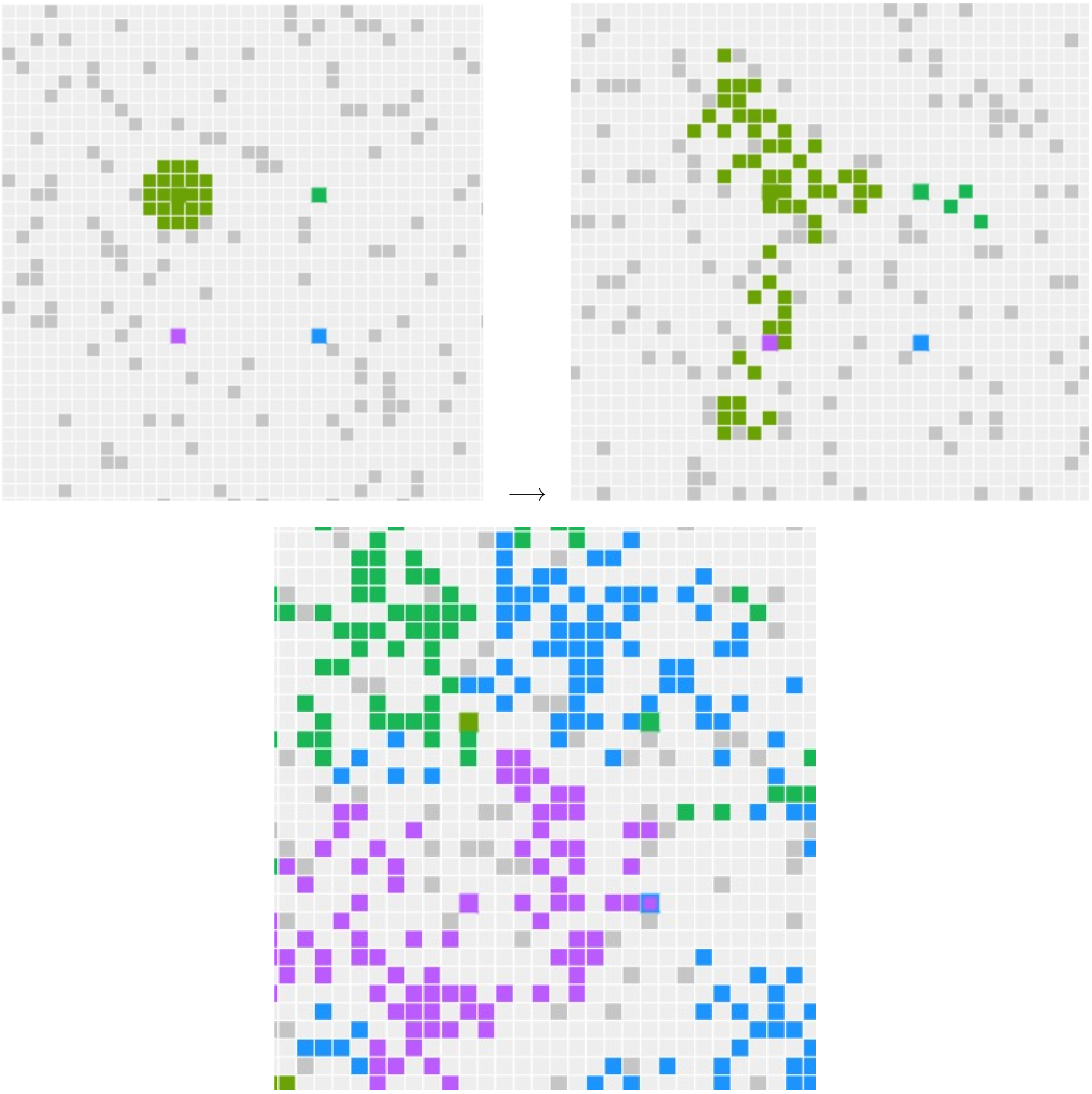
## Constraints

Scientific research communities are considered the most refined and valuable structures our civilization has produced. Significant numbers of specialists focused full-time on knowledge production led to the continuous growth of society. However, unlike every system, it does have systematic problems. And one way to view those obstacles is as network degradation.

Suppose we distinguish two ways of practising science: real science and careerist science. Real science is whatever habits and practises reliably produce knowledge. It is motivated by curiosity and characterised by honesty (Feynman: "I just have to understand the world, you see"). Careerist science, on the other hand, is driven by professional ambition and marked by alteration and scientific functionalities.

(Opinion). It may appear to be scientific, but it does not accord with the fundamentals. Careerists take up space in a real science research community, and they tie up the works. They promote themselves while the rest of the community is trying to learn and share what is true. Instead of aiming for clarity, they twist and muddle the knowledge in order to sound more impressive. They engage in what Harry Frankfurt would call scientific nuisance. And consequently, we might model them as dead nodes, immune to the good-faith information exchanges necessary for the growth of knowledge. Careerists certainly have the potential to stifle our scientific communities with fake knowledge.

♠ *Here,* ***grey*** *colored square-grid represents the presence of* ***careerists*** *(constraint)*



Of course, there is no definite boundary between careerists and real scientists. We all have a little careerism in us. The question is just how much the network can carry before going quiet!


## Acknowledgments

$\pi$ Kaushik Sanghani for suggesting me `http://35.161.88.15/interactive/going-critical/`, which is the main idea of my work.
$\pi$ Manish Mor, Bhaviya and Theophilus Gera for their thoughtful comments and suggestions.
$\pi$ Somanshu Kumar for suggesting me Claude Shannon's article.
$\pi$ Team AMaThing for publishing the article.


## Inspiration

$\pi$ All of Kevin Simler's work, especially `http://35.161.88.15`.
$\pi$ Grant Sanderson's work on `https://www.youtube.com/watch?v=gxAaO2rsdIs`.

# Real-life Applications of Optimization

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Knowingly or unknowingly, we are all using optimization in our daily lives in one way or another. So, it is a natural question to answer, What is optimization or what are the optimization problems? Optimization problems are the problems of finding the best (or optimum) solutions from all the possible (feasible) solutions, and the techniques for finding such solutions are termed optimization techniques. Based on the nature of the functions involved, the conditions on the constraint functions, and the number of objective functions, optimization problems may be categorised as linear or nonlinear, constrained or unconstrained, single-objective, or multiobjective.

In real-world problems, it is often required to deal with more than single objective and nonlinear functions; thus, most real-world problems fall into the category of nonlinear multiobjective optimization problems. A class of optimization problems in which two or more objectives are to be optimised subject to some constraints is known as a multiobjective optimization problem, and if the functions involved in these problems are nonlinear in nature, then these kinds of problems are called nonlinear multiobjective optimization problems.

For example, if you want to buy a scooter, you have two options: choose a scooter at a lower price and adjust for colour, or choose a scooter at a higher price and compromise on colour. These two objectives cannot be fulfilled simultaneously, and hence are called conflicting objectives. For these types of conflicting objectives, a compromise solution known as a Pareto optimal or Pareto efficient solution is proposed. This is just one simple example of optimization in our daily lives. Many miraculous applications in the world of optimization are constantly helping to make our lives easier.

Optimization includes an extensive variety of problems, including variational, continuous-time, optimal control, complex optimization, stochastic, fuzzy, and interval optimization. Each kind of problem has its own merits, which is why optimization problems have applications in diverse areas of business, including finance, the health sector, ecological problems, economics, the management sector, production and supply chain problems, and many more. Furthermore, the theoretical results of optimization are at the heart of many computer algorithms and engineering techniques. To sum it up, the following are some important applications of optimization problems:

1. Inventory management can be modelled as a multiobjective optimization problem of maximising profit subject to cost and space constraints.
2. Variational optimization problems are used to optimize the shape of aircraft and spacecraft subject to thickness, strain energy, or displacement bounds.
3. Stochastic optimization problems are the basic tools for the problems in the area of telecommunications.
4. Many numerical methods and algorithms based on the concept of optimization techniques are essential for solving ecological problems.
5. Financial problems such as portfolio optimization and risk management can be solved using fractional optimization.

These are just a few examples, but it is apt to say that “Optimization is everywhere!” Numerous applications of optimization problems are known, and there are still immense possibilities to explore and answer the question, “What can be the new area where optimization can potentially be applied?” To understand the depth of problems, one should continue to practise because, as William A. Dembski stated, “Constrained Optimization is the art of compromising between conflicting objectives,” and no art can be mastered in a day. So, keep searching, asking, practising, and attempting to make learning feasible while desiring compromised results, and you will undoubtedly get the best results!

# Natural Discussion of Uncertainty Quantification

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Uncertainty quantification is a field of study concerned with identifying, quantifying, and managing uncertainties in mathematical models and simulations. It involves analyzing the sources of uncertainty, characterizing the nature and magnitude of these uncertainties, and propagating them through the model to determine their impact on the output. Uncertainty can arise from various sources, including measurement error, model approximation, and variability in input parameters.

The goal of uncertainty quantification is to provide accurate and reliable predictions and assess the model's predictions' reliability by providing probabilistic bounds on the results. This information can be used to make informed decisions, manage risk, and prioritize future research efforts. Uncertainty quantification is used in a wide range of applications, including engineering, finance, environmental science, and healthcare.

Mathematical models and experimental data can both be affected by uncertainty in different ways. The causes of doubt can be divided into several categories, including:

## Parameter

The aforementioned predicament stems from model parameters that function as computer or mathematical model inputs. Yet, their precise values elude experimentalists and cannot be governed by physical trials or accurately deduced through statistical means. Instances of such difficulties include the local free-fall acceleration in falling object experiments, diverse material properties in finite element analyses for engineering purposes, and multiplier uncertainty in the macroeconomic policy optimization context.

## Parametric

The observed fluctuation in the model's input parameters can be attributed to its inherent variability. For instance, the dimensions of a manufactured workpiece may deviate from the prescribed specifications, leading to consequential variations in its overall performance.

## Structural uncertainty

Known as model inadequacy, model bias, or model discrepancy, this phenomenon arises due to insufficient knowledge regarding the underlying physics of the problem. It is contingent on how precisely a mathematical model captures the actual system in real-world scenarios, considering that models are usually only approximations of reality. A classic example is modelling a falling object using the free-fall model, which is inherently flawed due to the presence of air resistance. Even if all model parameters are known, a disparity is still expected between the model and actual physics.

## Algorithmic

Known as numerical uncertainty or discrete uncertainty, this category of uncertainty arises due to numerical errors and approximations incurred during the implementation of computer models. Most models are too intricate to be solved precisely, necessitating numerical techniques such as finite element or finite difference methods to approximate solutions to partial differential equations, which can introduce numerical errors. Examples of such approximations include numerical integration and truncation of infinite sums, which are essential in numerical implementations.

## Experimental

Known as an observation error, this type of uncertainty arises due to the inherent variability in experimental measurements. Experimental uncertainty is unavoidable and can be observed by repeating measurements multiple times with the same input settings and variables.

## Interpolation

This type of uncertainty arises due to the unavailability of adequate data collected from computer model simulations and/or experimental measurements. In cases where simulation data or experimental measurements are unavailable for certain input settings, one must resort to interpolation or extrapolation methods to predict corresponding responses.

## Aleatoric and epistemic

There are two main types of uncertainty, which are often seen in medical applications.

## Aleatoric

Aleatoric uncertainty, also known as stochastic uncertainty, pertains to unknown factors that vary each time an experiment is conducted. For instance, firing a single arrow with a mechanical bow that replicates each launch precisely in terms of acceleration, altitude, direction, and final velocity will not necessarily hit the same point on the target due to the random and intricate vibrations of the arrow shaft. The knowledge of these vibrations cannot be ascertained adequately to eliminate the resulting scatter of impact points. However, it is worth noting that the inability to measure this knowledge sufficiently using currently available devices does not necessarily imply the non-existence of such information, which would place this uncertainty in the category discussed below. The term “aleatoric” is derived from the Latin word “alea,” which means dice, alluding to games of chance.

## Epistemic uncertainty

Epistemic uncertainty, also known as systematic uncertainty, refers to the uncertainty that arises due to incomplete knowledge or understanding of the underlying processes or models that govern the system being studied. It arises due to various factors, such as measurement errors, model approximations, and missing data.

In the example you provided, the drag force acting on the object in the experiment is a source of epistemic uncertainty, as it is a known effect that is not accounted for in the commonly used model for gravitational acceleration. However, this uncertainty can be reduced by measuring the drag force and incorporating it into the model, resulting in a more accurate calculation of the gravitational acceleration.

Reducing epistemic uncertainty is important in many scientific and engineering applications, as it can lead to more reliable and accurate predictions and decisions. This can be achieved through better measurement techniques, more accurate models, and improved data collection and analysis methods.

## Types of problems

There exist two fundamental categories of challenges in the field of uncertainty quantification: firstly, the forward propagation of uncertainty, which involves the systematic propagation of various sources of uncertainty through a model to estimate the overall uncertainty in the system’s response. Secondly, the inverse assessment of model uncertainty and parameter uncertainty entails simultaneous calibration of model parameters utilizing test data. The former problem has attracted significant research and resulted in the development of numerous uncertainty analysis techniques. Conversely, the latter problem has garnered growing interest in the engineering design community due to the significance of uncertainty quantification in model development and the resulting predictions of the actual system response(s) in the creation of robust systems.

## Forward

Uncertainty propagation refers to the process of quantifying uncertainties in the output(s) of a system that result from uncertain inputs. This process focuses on the effects of parametric variability in the sources of uncertainty on the system outputs. The objectives of uncertainty propagation analysis may include the evaluation of low-order moments of the outputs, such as mean and variance, the assessment of the reliability of the outputs, which is particularly useful in reliability engineering, where the system’s outputs are closely linked to its performance, and the determination of the complete probability distribution of the outputs. The latter objective is especially relevant in the context of utility optimization, where the complete distribution is utilised to calculate the utility.

## Inverse

Given some experimental measurements of a system and some computer simulation results from its mathematical model, inverse uncertainty quantification estimates the discrepancy between the experiment and the mathematical model (which is called bias correction) and estimates the values of unknown parameters in the model if there are any (which is called parameter calibration or simply calibration).

Generally, this is a much more difficult problem than forward uncertainty propagation; however, it is vital since it is typically implemented in a model updating process. There are several scenarios of inverse uncertainty quantification:

- Bias correction
- Parameter calibration
- Bias correction and parameter calibration

# $\pi$: A Probabilistic Approach

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*$\pi$ is a famous and old number in mathematics. So old that its first-ever explanation dates back to the Bible in the form of a verse and is so well known that there is a day dedicated to it! Today, we have used computers to calculate the value of $\pi$ up to 22 trillion digits, but $\pi$ existed long before computers were invented. Ever wondered how people came up with techniques to calculate its value? What are some of the clever ideas that they used?*

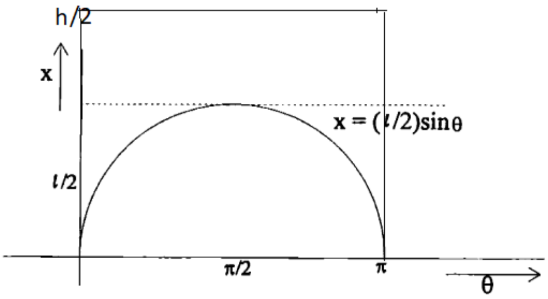
Archimedes made the earliest approximation of $\pi$ by using the concepts of geometry and algebra. Later, mathematicians started expressing $\pi$ in series such as the Wallis formula or the Gregory series.

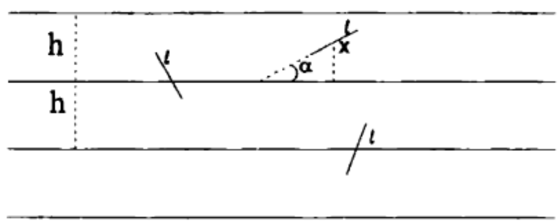
One such unique idea was given by a French scientist named Georges Buffon, who, being very fond of probabilities, did an experiment famously called “Buffon’s needle.” Accidentally, the probability of the event in the experiment came out to have an expression with $\pi$ in it!

- The experiment: Buffon randomly threw needles on a setup consisting of equally spaced parallel lines. The goal was to find the probability that a needle intersects a line.

- Assumption: The distance between lines is $h$, and the length of the needle is $l$, which is less than $h$ ($l < h$).

Mathematics is always incomplete without proof. Let us dive into its short and simple proof. For the first time in history, a geometric approach was used to find a probability, which makes it even more enjoyable. Probability is defined as desirable outcomes divided by the total number of possible outcomes. To find desirable outcomes, we need to define the position of the needle (when thrown) so that all the possibilities are unique. We take the parameters $\theta$ and x to describe it, where $\theta$ is the angle it makes with the horizontal and x is the perpendicular distance between the centre of the needle and the parallel line.




The target was to find the boundary condition equivalent to the case when the needle just touched the line. All the points lying under the curve of the boundary condition describe a unique position that a needle has when thrown randomly.

Counting all the points is equivalent to calculating the area under the curve, equal to $l$. To calculate the total number of possibilities, we deduce that $0<\theta<\pi$ and $0 < x < h/2$, the entire area is $\pi$h/2.

Finally, after taking the ratio, we get the probability as $\frac{2l}{\pi h}$. We got the $\pi$ term. Here is the result of a fun simulation I found on the internet, which shows the $\pi$ value calculated from an actual experiment:

| Measurement | Value |
| --- | --- |
| Needle Scale | 1 |
| Extent = Perimeter / Greatest Vertex Distance | 1 |
| Number of Drops | 5432 |
| Number of Hits | 3495 |
| Drops / Hits | 1.5542203147353362 |
| π ≈ 2 * Extent * Scale * Drops / Hits | 3.1084406294706723 |

In the above simulation, we got the figure 3.108, which largely deviates from the value of $\pi$ we know today. Another similar simulation on the internet showed that we needed to drop approximately 15000 needles to get the original value!

Long after Buffon experimented, various other versions of the experiment were tried; a few include the case where $l > h$ or the case where the needle was bent! Buffon's needle is one of the most eminent and unique experiments in the history of mathematics. He contributed innumerable things to a vastly unexplored area of geometrical probability. His attempts to connect mathematics with real-life experiments are much more fun to read.

# Going Up does not mean Going Down

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Algebra is an interesting field for researchers, yet it seems to be a dull field to others outside of their domain. Chain conditions on modules were conceptualised by Emmy Noether [4] in 1921. Noether proposed ascending chain conditions on commutative rings, which were later renamed Noetherian rings. In 1927, the idea of ascending chain conditions was dualized, and descending chain conditions were conceptualised by Emil Artin [1], which were later renamed Artinian rings.

In 1940, Hopkins Levitzki [3] proved that Artinian rings are Noetherian rings, but the converse may not hold.

**Theorem** ([2], Theorem 4.15). If $R$ is an artinian ring, it is also a noetherian ring, and $J(R)$ is nilpotent.

Now, coming to the application of this theorem. Consider a ladder on the wall. If a person wishes to ascend, this does not imply that he or she may descend, as this could result in an accident. If the person comes down, it means that he has already gone up the ladder and finished his work.

Analogous to the above application, consider the person to be a ring R and the right Artinian to be going down and the right Noetherian to be going up, which means that the physical phenomenon of going up and down is related to the Hopkins Levitzki Theorem.

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# Exchange Property

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Exchanging objects for money has been observed for a long time, and the practise was eventually termed the "barter system." It is rather tough to trade products with other non-relevant objects, such as trading a Samoyed puppy with a Turtle. We all know that a Samoyed puppy is worth far more than a Turtle. During one of these discussions, the intermediate value system, which we now know as the monetary value system, may have been formed.

Similarly, in algebra, we have various module structures in which we often check the exchange property. This is an important property that will also help us analyse the decomposition property of modules. The decomposition property decomposes modules into a direct sum of submodules. If each of the submodules involved in the direct sum is simple, we call this a "semi-simple module."

**Definition.** A module $M$ is said to have exchange property if it can be written in the form of $M = \oplus_\alpha M_\alpha$ where $M_\alpha \subseteq M$

We say that $M$ has finite exchange if $\alpha$ is finite and $M$ has full exchange if decomposition exist for any $\alpha$.

If each element of $R$ can be represented as sum of an unit and an idempotent, then we call $R$ to be clean ring. Clean rings were defined by Nicholson in [2]. There is a beautiful relationship between clean rings and exchange property of rings.

**Corollary** ([1], Corollary 12). A ring $R$ with no infinite set of orthogonal idempotents have the following equivalent properties.

1. $R$ is clean,
2. $R$ is exchange.

There can exists counterexamples for the conjecture "finite exchange implies full exchange" but as noted in ([3], §14) most of them are of free modules.

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# Neural Networks in Machine Learning

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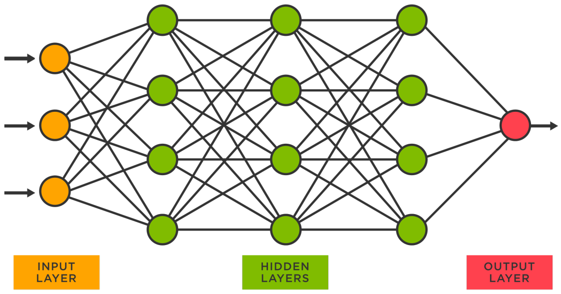
## Introduction

Neural networks are a potent tool in machine learning. The use of neural networks in machine learning has helped to solve a variety of challenges. They are inspired by the structure and characteristics of the human brain, and they are capable of learning complicated patterns in data. In this article, we will explore the basics of neural networks, including their structure, function, and how they can be trained to make predictions on new data.




## Structure of Neural Networks

A neural network is composed of layers of interconnected nodes, or neurons. The neurons in each layer receive input from the neurons in the previous layer, and they use this input to compute an output. The output of each neuron is then passed on as input to the neurons in the next layer. This manner continues till the output of the last layer is produced, which is the prediction of the neural network.

The first layer of a neural network is called the input layer, and it is responsible for receiving the input data. The closing layer of a neural community is referred to as the output layer, and it produces the predictions of the network.

The layers in between the input and output layers are referred to as hidden layers, and they are responsible for computing intermediate representations of the input data.

## Function of Neural Networks

The neurons in a neural network compute their output using a mathematical function called an activation function. The activation function takes the weighted sum of the inputs to the neuron and applies a non-linear transformation to produce the output. This non-linear transformation is what allows neural networks to learn complex patterns in the data.

The weights in a neural network are learned during the training process, which involves feeding the network a set of labeled data and adjusting the weights to minimize the difference between the network's predictions and the true labels. This is generally achieved using an optimization algorithm, such as stochastic gradient descent.

## Types of Neural Networks

There are several types of neural networks, each with its own structure and function. Here are a few examples:

1. Feedforward Neural Networks (FNNs): This is the most simple kind of neural network, the place the data flows in one direction, from the input layer to the output layer.

2. Convolutional Neural Networks (CNNs): This type of neural network is used for image recognition and processing. It uses convolutional layers to extract features from the input image.
3. Recurrent Neural Networks (RNNs): This type of neural network is used for sequential data, such as speech or text. It uses recurrent layers to store information about the previous inputs and make predictions about the future inputs.

## Applications of Neural Networks

Neural networks have been used to solve a huge range of problems, including photo recognition, speech recognition, natural language processing, and game playing. Here are a few examples:

1. Image Recognition: Neural networks have been used to classify images into different categories, such as cats and dogs, or to detect objects in images, such as cars and pedestrians.
2. Speech Recognition: Neural networks have been used to transcribe spoken words into text, or to recognize the speaker of a voice.
3. Natural Language Processing: Neural networks have been used to perform tasks such as sentiment analysis, language translation, and text generation.

## Conclusion

Neural networks are a potent tool in machine learning. The use of neural networks in machine learning has helped solve a variety of challenges. They are inspired by the structure and characteristics of the human brain, and they are capable of learning complicated patterns in data. In this article, we have explored the basics of neural networks, including their structure and function and how they can be trained to make predictions on new data.

# Markov Chains: A Non-Independent Stochastic Process

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Markov chains, named after Russian mathematician Andrei Andreevich Markov, are a non-independent stochastic process in which previous events are irrelevant and future events are solely dependent on the present. The origin of chains goes back to the study of the weak law of large numbers on an independent sequence of random variables by Russian mathematician Pafnuty Chebyshev and his students Markov and Aleksandr Mikhailovich Lyapunov. The papers authored by Mathematician Pavel Nekrasov had a huge impact on the origin of Markov Chains. The paper published in 1902 by Nekrasov stated that the "necessary condition for weak law of large numbers is the independence of the random variables". But Markov disagreed with the statement, which led to the development of Markov Chains. Markov published the first paper on the dependency of chains where the weak law of large numbers holds, which was later renamed as the Markov Chain. Later on, Markov Chains had huge applications in different fields of Mathematics, Probability, Statistics, and Physics.

A Markov chain is the representation of the system that transits from one state to another with known or unknown transition probabilities. At first, the Markov chains are defined as discrete states with discrete time distributions. Few years after the introduction of Markov Chains, Soviet mathematician Andrey Kolmogorov developed the discrete space Markov Chain with continuous time distributions. This categorises Markov Chains into Continuous Time Markov Chains (CTMC) and Discrete Time Markov Chains (DTMC). Both types can be viewed in a real-world scenario. A few examples of DTMC are Druncard's Walk, the Gambler's Ruin Problem, the Epidemic Model based on chain dependency, etc. A few examples of CTMC are the birth-death process, stock prices, etc.

## Representation of Markov Chain

Consider the state space $S = \{1, 2, 3, \cdots\}$, with transitional probabilities $p_{i\to j}\ \forall\ i, j \in S$. Let $X_k$ be the state at time $k$, then we have the conditional probability

$$P(X_k = j \mid X_{k-1} = i_{k-1}, X_{k-2} = i_{k-2}, \cdots, X_1 = i_1) = P(X_k = j \mid X_{k-1} = i_{k-1})$$

$\forall\ j,\ i_{k-1},\ i_{k-2},\ \cdots,\ i_1 \in S$ which represents the Markov property i.e., the transition into future state $(X_k)$ depends only on the current state $(X_k)$ but it is independent of past states $(X_{k-1}, X_{k-2}, \cdots, X_1)$.

## Applications

Markov chains have a considerable number of applications in various fields of science and technology. One such application can be observed in search engines, where it uses the concept of chains. Google's search engine uses Markov chains to suggest items according to present internet activity. Queueing models are another type of application where the number of customers that are in the system serves to express the status of the system. Monte Carlo Markov Chain (MCMC) is a kind of sampling technique that uses the basic concepts of Markov properties.

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# How did the Königsberg Bridge Problem Change Mathematics?

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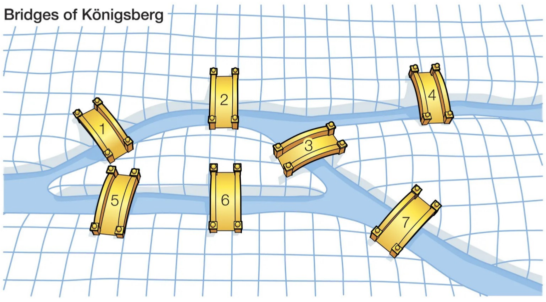
On any modern map, Konigsberg would be difficult to locate, but thanks to a peculiar feature of its location, it has become one of the most well-known places in mathematics. It is present on either side of the Pregel River and was a medieval German city. There were two sizable islands in the middle, and seven bridges linked the two islands to the river banks as well as to one another.

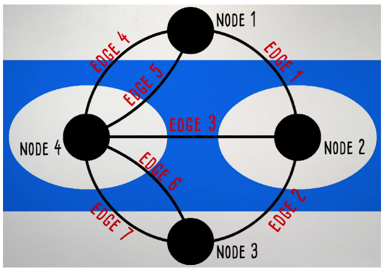
**Königsberg Bridge Problem.** How can one traverse through all seven bridges only once without repeating any of them?

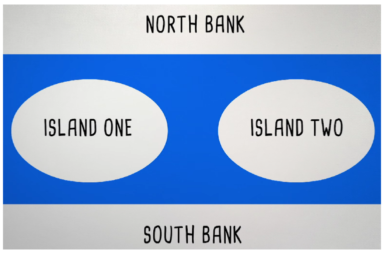



Just take a moment to think about it. Have you surrendered? You should absolutely do so. It is just not conceivable.

Before analysing the solution, let us delve into the history of how it all began and the origin of Euler's solution. The story begins with Leonhard Euler, a prominent mathematician who created a new branch of mathematics called the Geometry of Position, which is now widely known as Graph Theory. This novel type of geometry did not exist before Euler introduced it.

In his famous paper from 1736, Euler addressed the Königsberg bridge puzzle, which ultimately gave birth to the field of graph theory. He realised that the order of crossing the bridges did not matter in finding the solution. He simplified the map by representing the four land masses as nodes connected by edges to depict the bridges. Using this simplified graph, it became easy to calculate the degree of each node, which is the number of bridges that each land mass crosses.



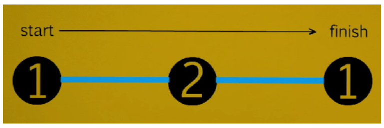




Why are degrees important? The significance of degrees in the context of this challenge is that in order to traverse between landmasses, one bridge must be crossed to enter and another to exit. Similarly, while visiting each landmass, the number of bridges connected to it must be an even number. This is because, for any route, the bridges connecting each node must be paired in a unique manner, with the exception of the starting and ending points. The graph demonstrates that all four nodes have an odd degree, which means that at some point, a bridge will need to be crossed twice, regardless of the path taken.

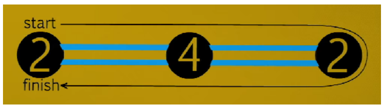
Euler formulated a general theory applicable to all graphs containing at least two nodes based on the insights gained from this demonstration.

There are only two possible scenarios in which a path that traverses every edge exactly once, known as an Eulerian path, can exist.

- The first one is when there are precisely two nodes with an odd degree, while all other nodes have an even degree. One of the odd nodes will be the starting point, while the other will serve as the endpoint.




- Another scenario where an Eulerian path can exist is when all nodes in the graph have the same degree, leading to what is known as an "Eulerian circuit" where the path both starts and ends at the same node.




As we go into the second case, let us go over some terminology.

**Definition** (Walk). A walk in a graph $G$ is defined as a finite alternating sequence of vertices and edges of the form: $v_0$, $e_1$, $v_1$, $e_2$, $v_2$, $e_3$, $\cdots$, $v_{n-1}$, $e_n$, $v_n$.

If $v_0 = v_n$, then the walk is called a closed walk. Else, the walk is called an open walk.

**Definition** (Circuit). A closed walk in which no vertex (except its terminal vertices) appears more than once is called a circuit.

**Definition** (Connected Graph). A graph $G$ is said to be connected if there is at least one path between every pair of vertices.

**Definition** (Euler Graph). A closed walk running through the edge of a graph exactly once is known as an Euler line, and a graph consisting of an Euler line is called an Euler graph.

The following theorem is essential in determining whether a graph is an Euler graph.

**Theorem**. A given connected graph $G$ is an Euler graph if and only if all vertices of $G$ are of even degree.

**Proof**.

Part I Let $G$ be a connected Euler graph, we have to show that all the vertices of $G$ are of even degree. Since $G$ is an Euler graph, then by definition of an Euler graph, it contains an Euler line, which is a closed walk running through every edge of a graph $G$ exactly once. Now at any vertex $v$ of $G$, we enter through one edge and exit through the other edge while touching each vertex. During each touch, we have two distinct edges incident to vertex $v$, so $v$ is of even degree.

If $v$ is the initial vertex, then an edge incident to it is also used to start, and another edge is used to finish as the walk is closed. Hence the initial vertex is also of an even degree. Hence, each vertex is of an even degree.

Part II Let $G$ be connected, and each vertex of $G$ be of even degree, we have to show that $G$ is an Euler graph.

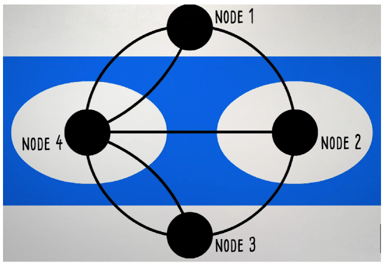
Since $G$ is connected and all the vertices of $G$ are of even degree. Let $v$ be any vertex of $G$, construct a closed walk $W_1$ starting from $v$ tracing every edge in the way exactly once and reaching again. Each vertex is of an even degree, so this will be possible. If all the edges of $G$ are traced, then $G$ is an Euler graph. If not, then let $g_1$ be the graph consisting of the closed walk $W_1$ and $g_1'$ be the graph consisting of all the edges of $G$ which are not in $W_1$.

Clearly, $g_1$ and $g_1'$ are two subgraphs of $G$, as $G$ is connected, there exists at least one vertex common to both $g_1$ and $g_1'$ say $u$.

Now start from $u$ to construct a closed walk $W_2$ in $g_1'$ which is also possible as each vertex of $g_1'$ is also of even degree. When we combined $W_1$ and $W_2$, we form a new closed walk $W_3$.

If $W_3$ is $G$ then $G$ is an Euler graph. Otherwise, we can proceed in a similar way to form a new closed walk form in the subgraph of $G$, which consists of those edges that are not in $W_3$. This process terminates after certain stages, and the final closed walk will be $G$.

Hence $G$ is the Euler graph.




Upon examining the Königsberg Bridges graph, it becomes apparent that not all vertices possess an even degree. Consequently, this graph does not meet the criteria for an Euler graph. As a result, it is impossible to traverse each of the seven bridges only once and arrive back at the starting point.

During World War II, the Soviet Air Force destroyed two of Königsberg's bridges, which made it possible to create an Eulerian path through the remaining bridges. Despite the city's destruction, it is remembered in history for the puzzle that led to the creation of a new branch of mathematics. Königsberg was eventually rebuilt as the Russian city of Kaliningrad, and the seven bridges that were once part of the puzzle are no longer standing.

**How can an Eulerian path be created in Königsberg?** It is quite straightforward - all we need to do is remove a single bridge. Interestingly, history has already provided an example of an Eulerian path in Königsberg.

Leonhard Euler's solution of the Königsberg Bridge Problem in 1736 using a graph is widely considered the most renowned application of graph theory. This long-standing problem was finally solved with the help of Euler's new theory of graphs and topology, which he established with his first published paper in this field.

# Importance of Mathematical Modeling

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In the real world, we are often interested in predicting the values of variables and parameters like population predictions, the number of infected persons by diseases, the speed of moving objects, the concentration of the substance in a compartment, etc. Mathematical modelling has the ability to predict variables and parameters from the past as well as real-time data of the system, which can help us understand better behaviour or aid us in planning for the future. A mathematical model can reflect or mimic the behaviour of a real-life situation, and we can get a better understanding of the system through proper analysis of the model using appropriate mathematical tools. Mathematical modelling has great importance in physics, chemistry, biology, engineering, economics, and even industry. For example, if we consider mathematical modelling in the manufacturing industry, many aspects of manufacture, from mining to distribution, are susceptible to mathematical modelling. In fact, manufacturing companies have participated in several industrial mathematical workshops, where they discussed various problems and obtained solutions through mathematical modelling. To describe the real-world problem and investigate the questions that usually arise from it, mathematical modelling is a very important tool of mathematics. Using these tools, a real-world problem of the system is translated first into the logical structure of the system and then converted into appropriate mathematical structures that represent the real-world problem. After converting the real-world problem into mathematical form, the solutions to the mathematical model are obtained by various solution methods, which are then interpreted in the language of the real-world problem to make predictions or understand the situation for taking the right decisions for the system. Mathematical modelling deals with problems from biology, chemistry, engineering, ecology, the environment, physics, the social sciences, statistics, wildlife management, etc., and helps biologists, chemists, ecologists, and economists analyse the problems of the system. Mathematical modelling helps them undertake experiments on the mathematical representation of a real-world problem instead of undertaking experiments in the real world.

Mathematical models can be used to help with all kinds of system decisions. In today's complex and fast-moving environment, firms may have a wide variety of strategic and operational choices. A mathematical model helps system persons explore complex choices, using sets of assumptions to represent alternative future operating environments. It also helps to develop a clearer understanding of the inherent pattern of relationships between the variables and the likely outcomes. In the end, it is the judgement of the decision-makers that is crucial, but a well-designed model can make the exercise of that judgement easier. A model can help with all three stages of decision-making: analysis, choice, and implementation. To understand the real potential of an opportunity, a model should be constructed in a way that will allow the impact of alternative assumptions and scenarios to be explored. Through the flexing of assumptions and the methodical examination of alternatives, the range of potential outcomes is revealed. Identifying the extent of this range of outcomes enables the model user to understand the potential risk and reward of the whole opportunity.

It also took time for the importance of mathematical modelling to be completely understood. Physics and its application to nature and natural phenomena is a major force in mathematical modelling and its further development. Later, economics became another area of study where mathematical modelling began to play a major role. If the model has been built with sufficient detail, such that it shows each of the variables and parameters of the system accurately, then such a mathematical model will always provide early warnings of unforeseen problems to run the system properly, which helps to improve the decision-making process in the future.

Now-a-days, we have different kinds of branches of mathematics like graph theory, discrete mathematics, calculus, operations research, computing, fractional calculus, etc., so people can develop mathematical models by using any one of the fields, like computer lab design, where they can utilise a graph theory-based model. To predict the populations or bacterial growth, they utilised calculus, interpolation, or computational techniques. To find pollutant in rivers, they utilised statistical models or compartment modelling. A compartmental model is used to understand the effects of drugs on the body. To find the velocity of the moving object, another calculus-based model is utilised. To find maximum profit or to complete the work in the minimum amount of time with the minimum amount of risk and the maximum quality of service, the operations research-based model is utilised. The origin of mathematical modelling is in the year 2000 BC, when the three ancient civilizations of Babylon, Egypt, and India had a good knowledge of mathematics and used mathematical models in various spheres of life. In the field of astronomy, Ptolemy, influenced by Pythagoras' idea of describing celestial mechanics by circles, developed a mathematical model of the solar system using circles to predict the movement of the sun, the moon, and the planets. The model was so accurate that it was used until the early seventeenth century, when Johannes Kepler discovered a much more simple and superior model for planetary motion in 1619. This model, with later refinements done by Newton and Einstein, is still in use today. In the Western world, it was only in the sixteenth century that mathematics and mathematical models developed. The greatest mathematician in the Western world after the decline of Greek civilization was Fibonacci, Leonardo da Pisa. The son of a merchant, Fibonacci made many journeys to the Orient and familiarised himself with mathematics as it had been practised in the Eastern world. He used algebraic methods to improve his trade as a merchant. He first realised the great practical advantage of using the Indian numbers over the Roman numbers, which were still in use in Europe at that time.

The developments of Mathematical models are also noticed in the field of biology specifically in cell geometry and process of cell, in the field of engineering like the study of variation of shielding gas in GTA welding, in the field of agriculture for prediction of aging behavior for Al-Cu-Mg/ Bagasse particular composites and to predict sunflower oil expression, in the field of social sciences for public health decision making and estimations, in other field too like for developing of cerebral cortical folding patterns which have fascinated scientists with their beauty and complexity for centuries, in the development of a new three dimensional mathematical ionosphere model at the European Space Agency/European Space Operators Centre, in battery modeling or mathematical description of batteries, which plays an important role in the design and use of batteries, estimation of battery processes and battery design.

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